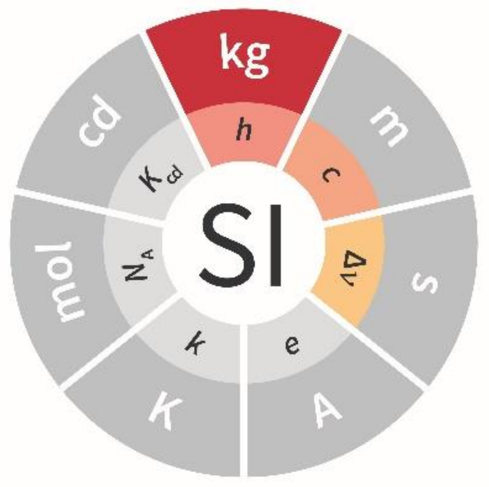
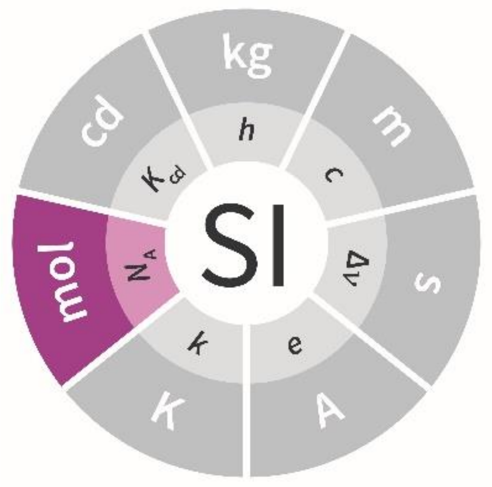
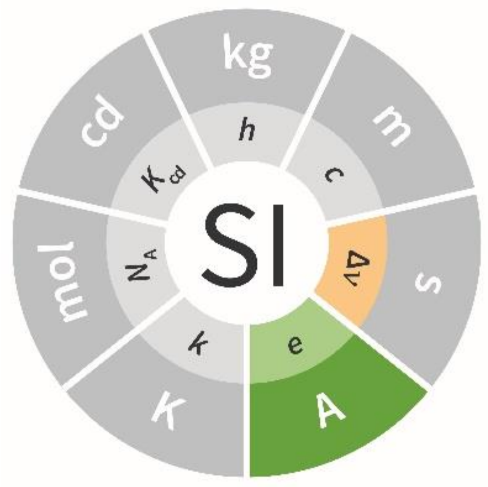
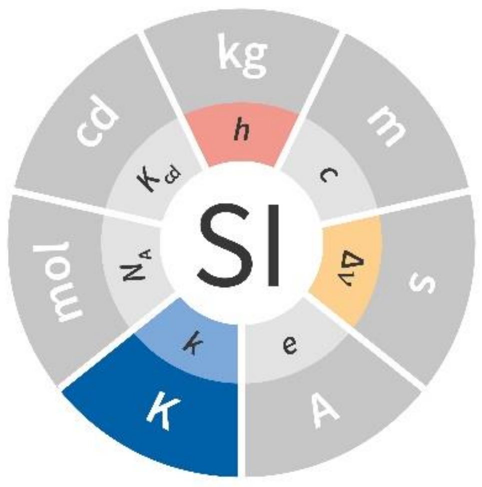
New definitions: kg, mol,Ampere,Kelvin

Dr. Kanokwan Nontapot

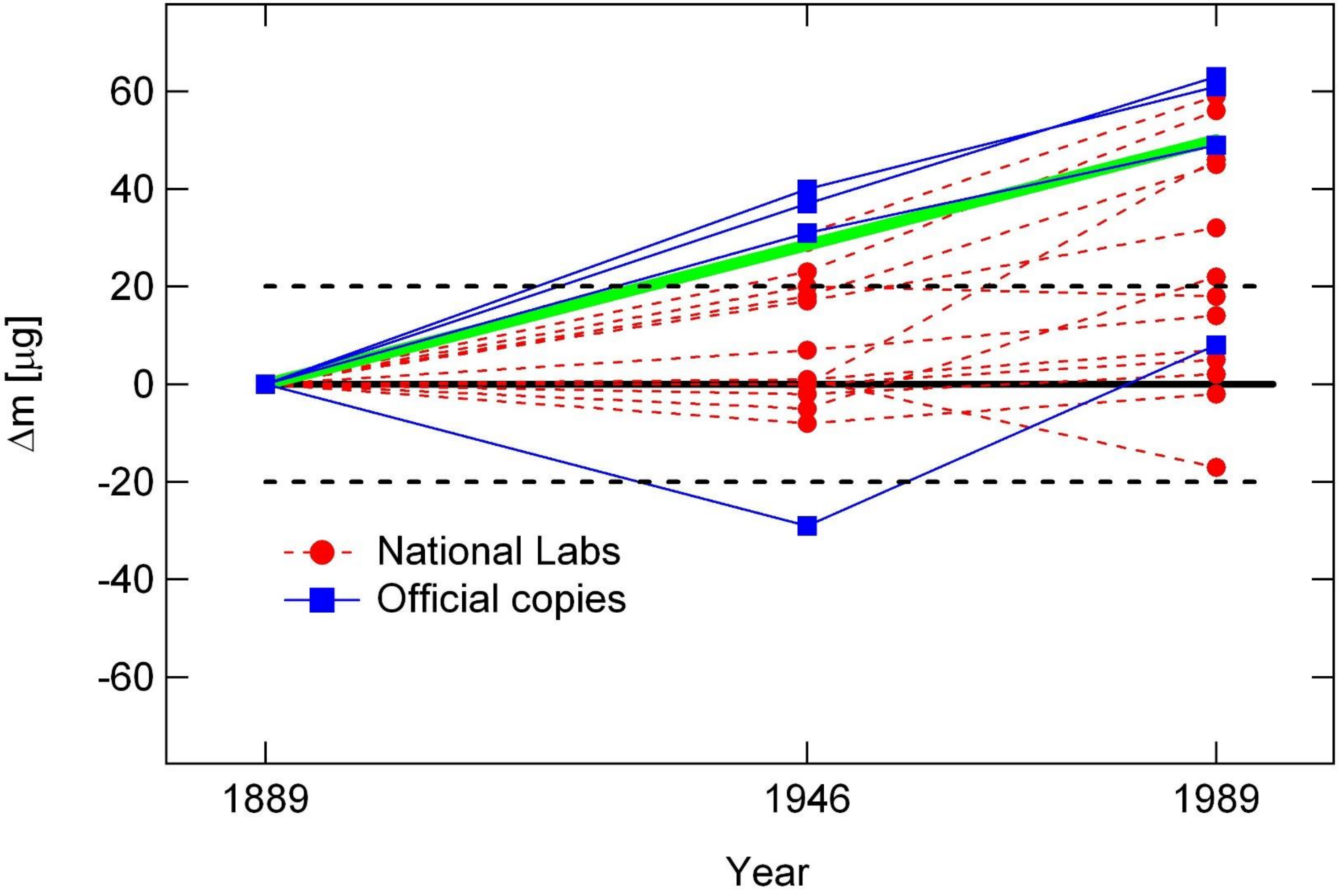








The Last Artefact: IPK



Mass and Energy

Energy is Mass and Mass is Energy

$$E=mc^2$$

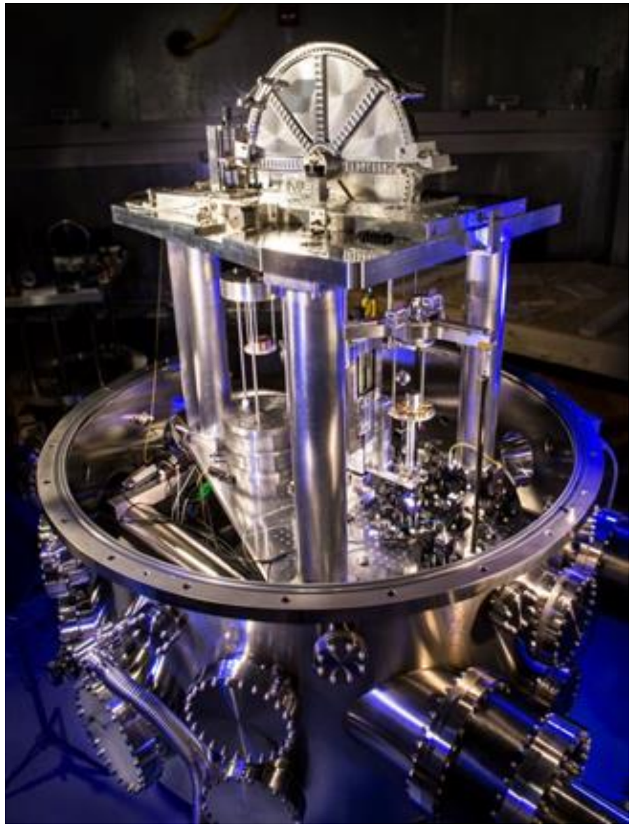
Energy of a photon

$$E=hv$$

$$hv=mc^2$$

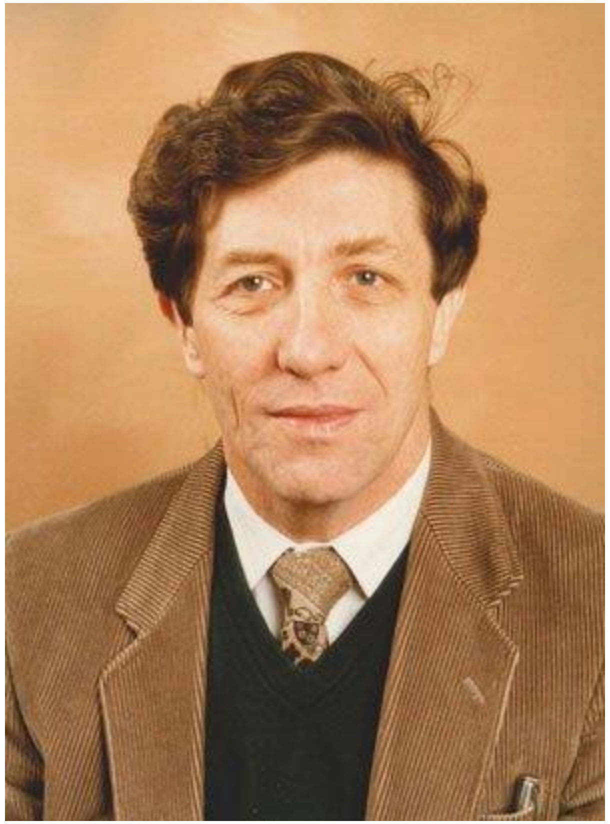
How can we find *h* ??

Kibble balance

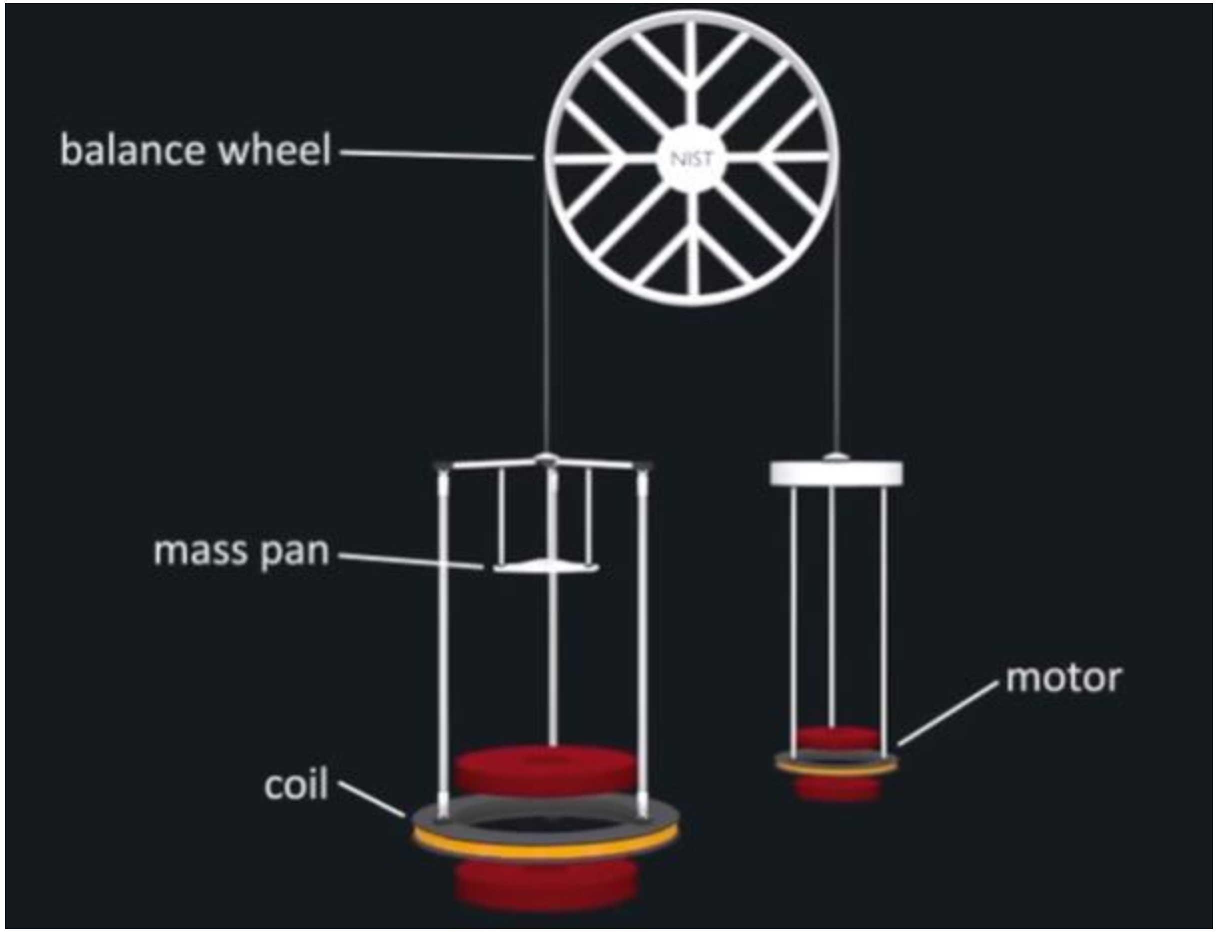


Avogadro project

Kibble Balance (Watt Balance)

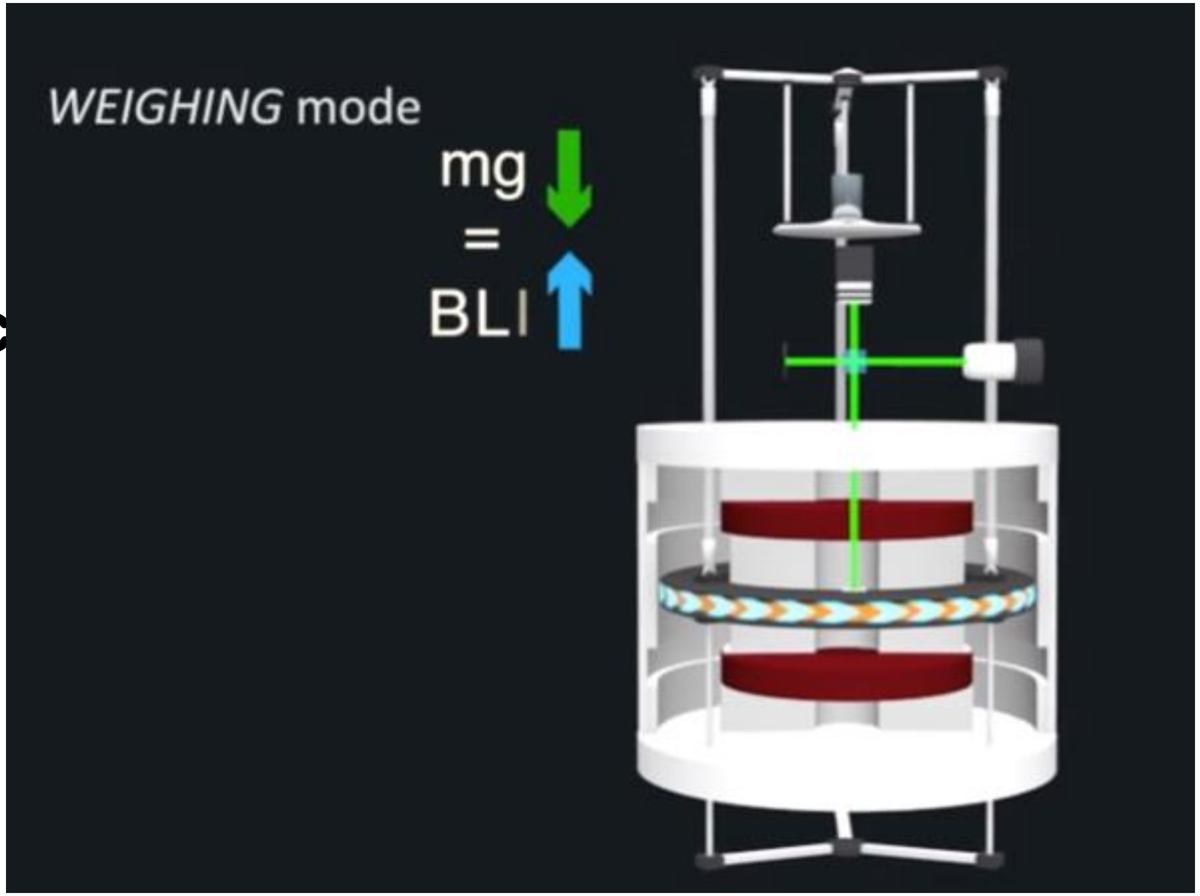


Bryant Kibble, NPL
(1938-2016)



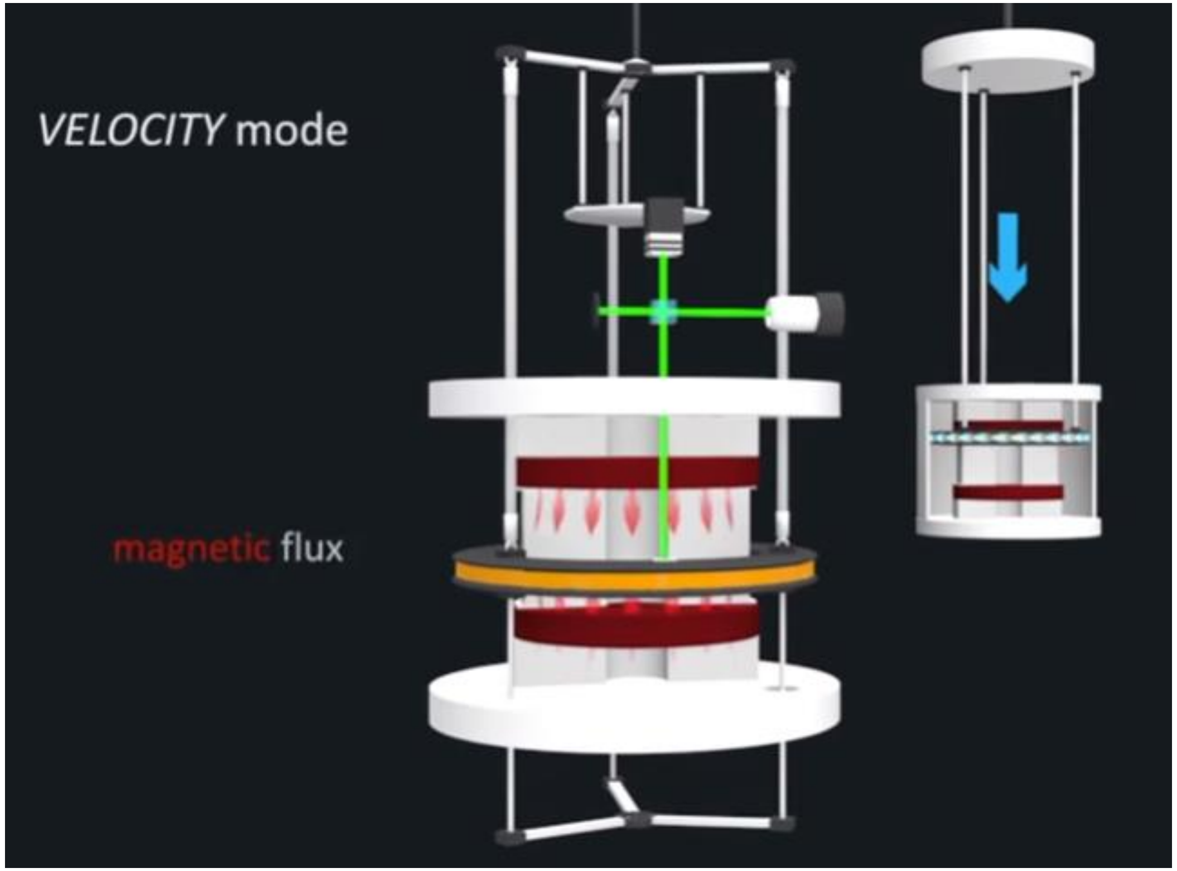
Kibble Balance

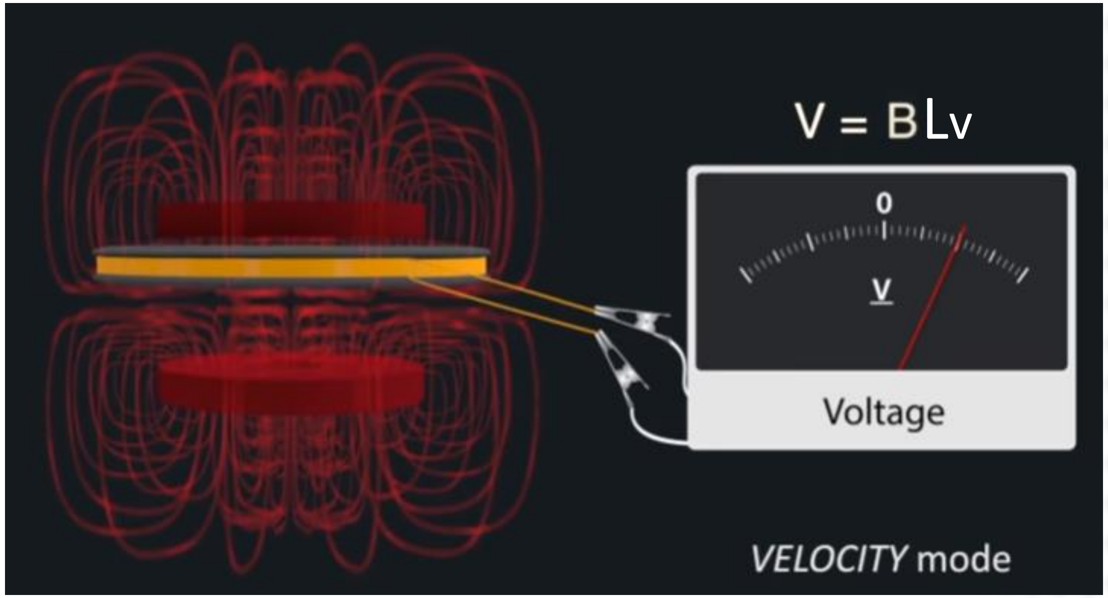
- Weighing mode
- B and L are REALLY diffic
  measure!



Kibble Balance

- Velocity mode





Kibble balance

- **Weighing mode** $$mg = IBL$$ $$BL = \frac{mg}{I}$$
- **Velocity mode** $$V = vBL$$ $$BL = \frac{V}{v}$$

$$\frac{V}{v} = \frac{mg}{I}$$

Mechanical power (Watt)

$$mgv = IV = \frac{V^2}{R}$$

Electrical power(Watt)

Watt Balance ☺

But….how can we related to a plank constant (*h*) ??

$$mgv = IV = \frac{V^2}{R}$$

Voltage: Josephson effect

$$K_J = \frac{2e}{h}$$

Resistance: Quantum Hall effect

$$R_K = \frac{h}{e^2}$$

$$\frac{hf}{2e} \quad \frac{hf}{2e}$$

$$mgv = \frac{VV}{R}$$

$$\frac{h}{e^2}$$

$$h \propto m\frac{gv}{f^2}$$

Avogadro Project (X-ray crystal density method) since 1990s

PTB in Germany, NMIJ in Japan, NIM in China, METAS in Switzerland, NIST in the U.S., INRiM in Italy, BIPM in France, and IRMM in Belgium..

- Counting Si_{28} atoms in a perfect sphere of Si_{28} ball weight around 1 kg
- Raw material worth ~ 1 million Euro

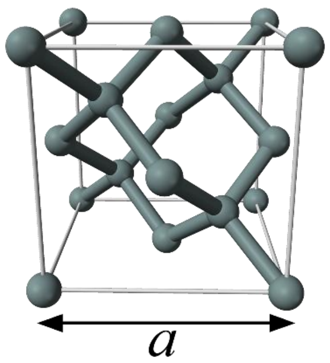
Avogadro project

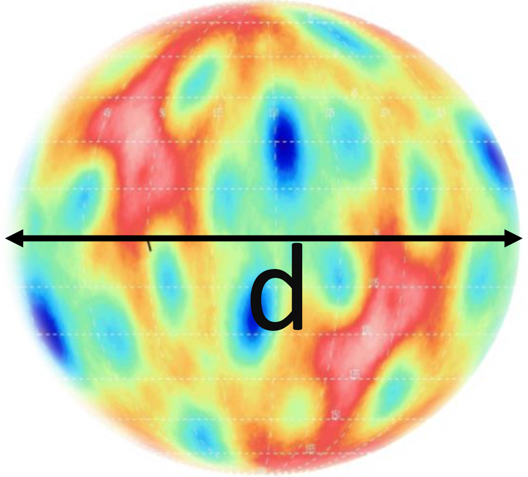
- The size of a select sphere would be measured using optical interferometry to an uncertainty of about 0.3 nm on the radius—roughly a single atomic layer.
- The precise lattice spacing between the atoms in its crystal structure (≈ 192 pm) would be measured using a scanning X-ray interferometer. This permits its atomic spacing to be determined with an uncertainty of only three parts per billion.
- With the size of the sphere, its average atomic mass, and its atomic spacing known, the required sphere diameter can be calculated with sufficient precision and low uncertainty to enable it to be finish-polished to a target mass of one kilogram

Counting Si atoms

- **Number of atoms**

$$n = 8\frac{V_{\text{Sphere}}}{V_{\text{unit cell}}} = \frac{4\pi}{3}\frac{d^3}{a_0^3}$$





d_{sphere}= 93 710 811.21(50)nm

- **By weighing the sphere, we can determine the Avogadro constant.**
- **Mass** $$m = N \times \frac{m_{\text{Si}}}{m_e} \times m_e = \frac{4\pi d^3}{3a_0^3} \times (f_{28}r_{28} + f_{29}r_{29}f_{30}r_{30}) \times \frac{2hR_\infty}{\alpha^2 c}$$

$$hN_A = \frac{cA(e)_r M_u \alpha^2}{2R_\infty}$$

Acceptance criteria for redefinition

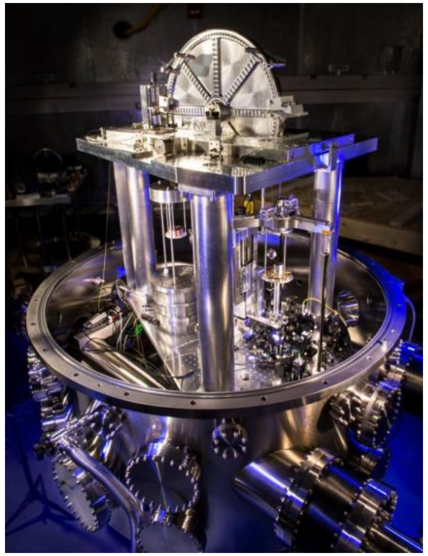
- For the redefinition of the kilogram, at least three separate experiments be carried out yielding values for the Planck constant having a relative expanded (95%) uncertainty of no more than 5×10^{-8} and at least one of these values should be better than 2×10^{-8}. Both the Kibble balance and the Avogadro project should be included in the experiments and any differences between these be reconciled.
- For the redefinition of the kelvin, the relative uncertainty of Boltzmann constant derived from two fundamentally different methods such as acoustic gas thermometry and dielectric constant gas thermometry be better than 10−6 and that these values be corroborated by other measurements.

Planck and Avogadro

Before m_K= exactly 1 kg

After m_K= 1 kg ± 1.2x 10^{-8}kg

Planck's Constant



before

after

Avogadro Number

before

after

$$hN_A = \frac{cA(e)_r M_u \alpha^2}{2R_\infty}$$

Before h = 6.6260XXXX ×10^{-34} kg·m²·s⁻¹

After h = exactly 6.62607015 ×10^{-34} kg·m^2·s^{-1}

Before N_A = 6.022140XX×10^{23} mol^{-1}

After N_A = exactly 6.02214076×10^{23} mol^{-1}

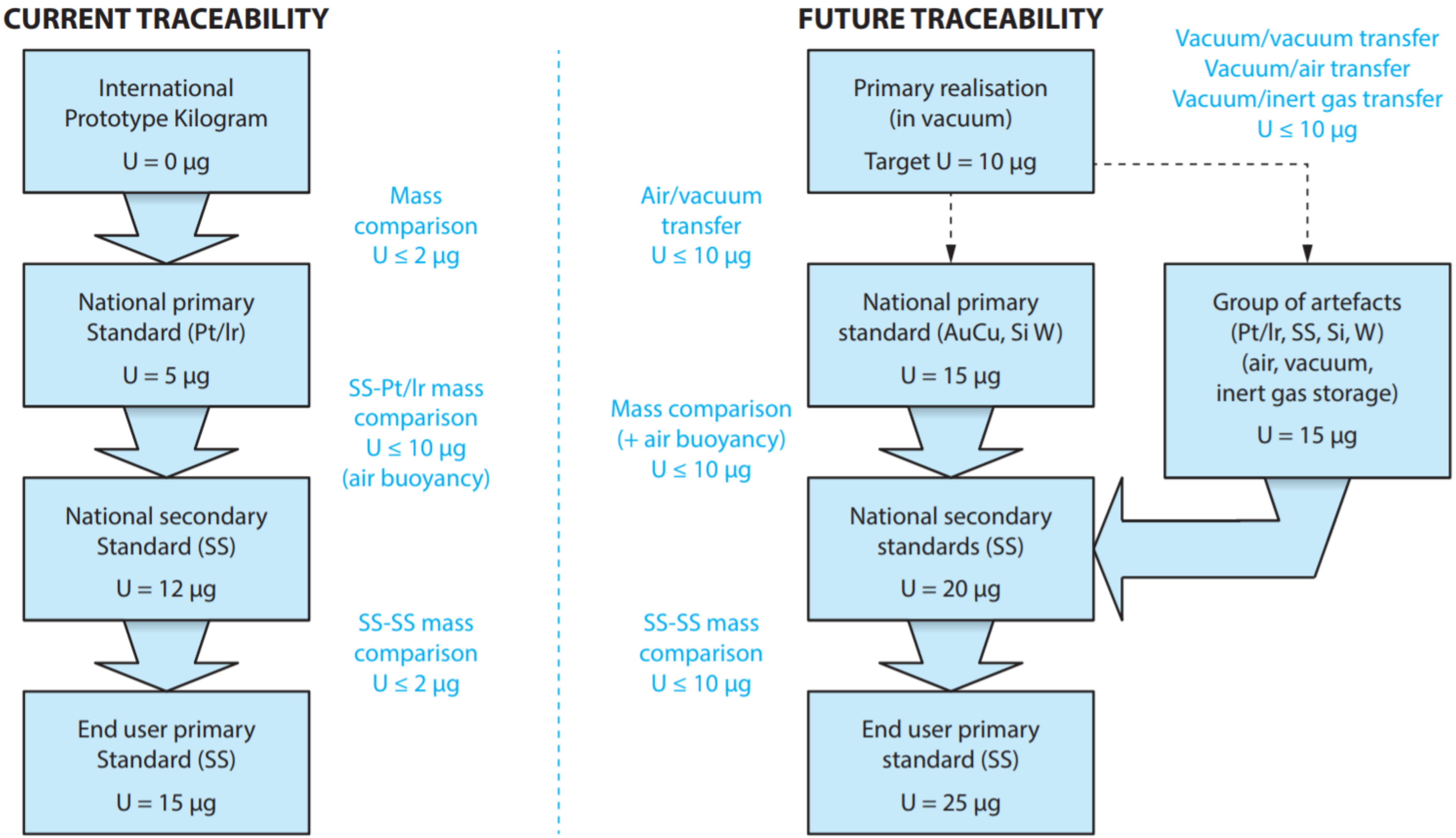
SI mass unit: kilogram

- **Old:** The kilogram is equal to the mass of the International Prototype Kilogram.
- **New:** The kilogram (kg) is defined by taking the fixed numerical value of the Planck constant h to be 6.626,070,150 × 10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^1, where the metre and the second are defined in terms of *c* and Δv.
- **Translation:** The kilogram will be defined in terms of Planck’s constant instead of the mass of a cylinder of metal called the International Prototype Kilogram.

SI amount of substance unit: mole

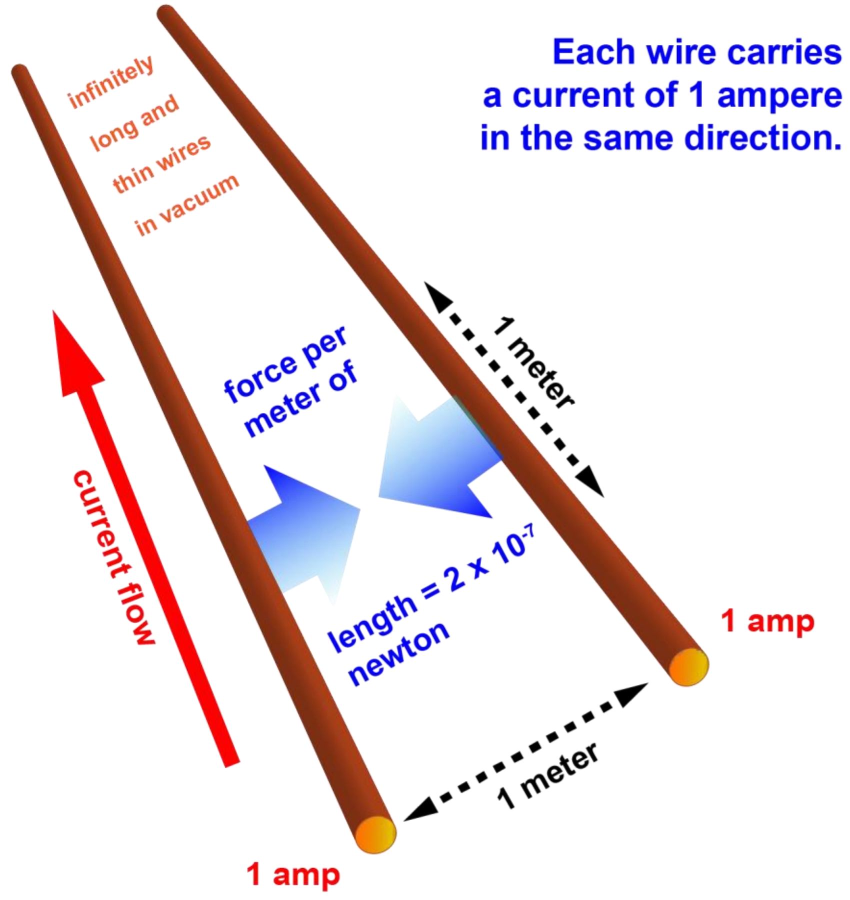
- **Old:** The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kg of carbon-12.
- **New:** The mole (mol) contains exactly 6.022,140,76 × 10^{23} elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number.
- **Translation:** The mole will be defined in terms of a specific number of atoms or molecules, rather than by a quantity intimately connected to measuring the mass of a sample.

Effect of the new kilogram



Ampere

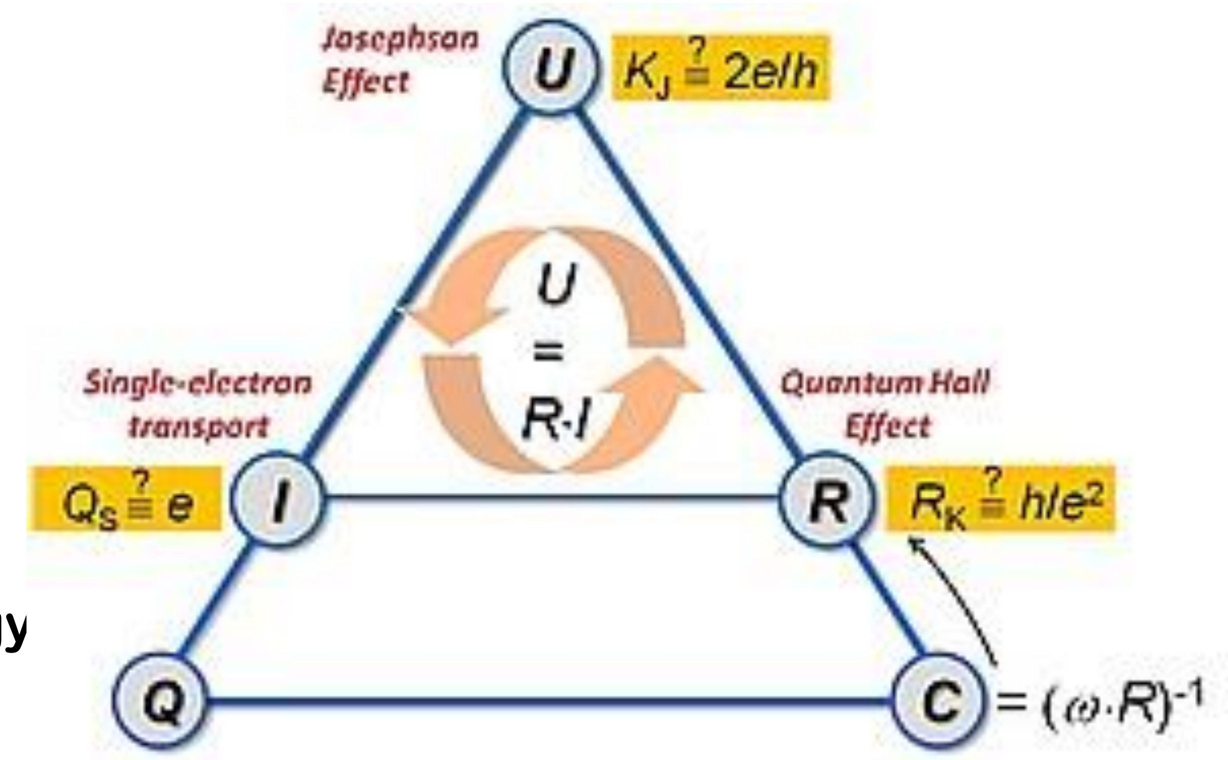
The present-day SI definition follows this arrangement. If it were set up under ideal conditions with the wires exactly 1 meter apart, a current of 1 ampere would result in a force between the wires of 2 X 10-7 newtons. That's not much — roughly a ten-millionth of the weight of an average apple.



The present SI definition of the ampere is based on André-Marie Ampère's famous 200-year-old experiment.

Ampere problem

- Josephson effect (1962) (V)
- Quantum hall effect (1980) (R)
- Calculate I from V=IR ampare's law
- Uncer 10 times better than conventional 'force based' metrology
- So we "Redefinition" in term of "e"

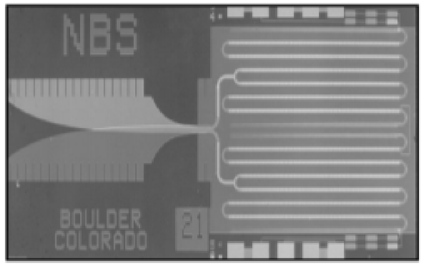


Quantum Metrology Triangle (Ohm's law)

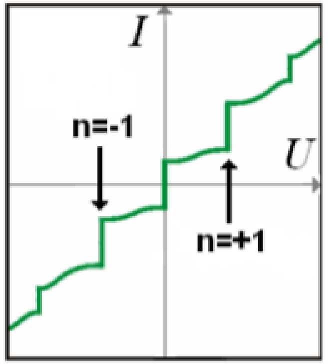
When the ampere is redefined in terms of e, all three components of Ohm's law will finally be formally interconnected by only two fundamental constants: h and e. Scientists will be able to use the Quantum Metrology Triangle (Ohm's law in quantum terms) to check the values of current, voltage and resistance by using any two units to determine the value of the third.

Quantum Metrology

Josephson effect

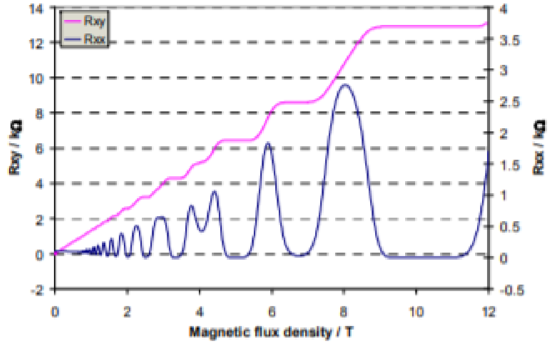


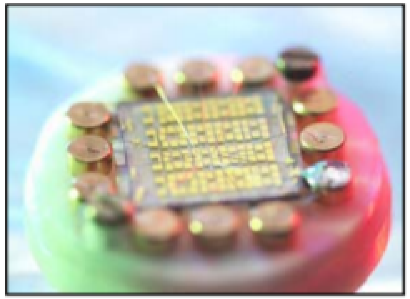
NIST / Wikimedia Commons



$$U_J = n\frac{f}{K_J}, \qquad K_J = \frac{2e}{h}$$

Quantum-Hall effect





$$R_H(i) = \frac{R_K}{i}, \qquad R_K = \frac{h}{e^2}$$

Elementary Charge

The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge *e* to be $1.602\ 176\ 634 \times 10^{-19}$ when expressed in the unit C, which is equal to A·s, where the second is defined in terms of $\Delta\nu_{Cs}$.

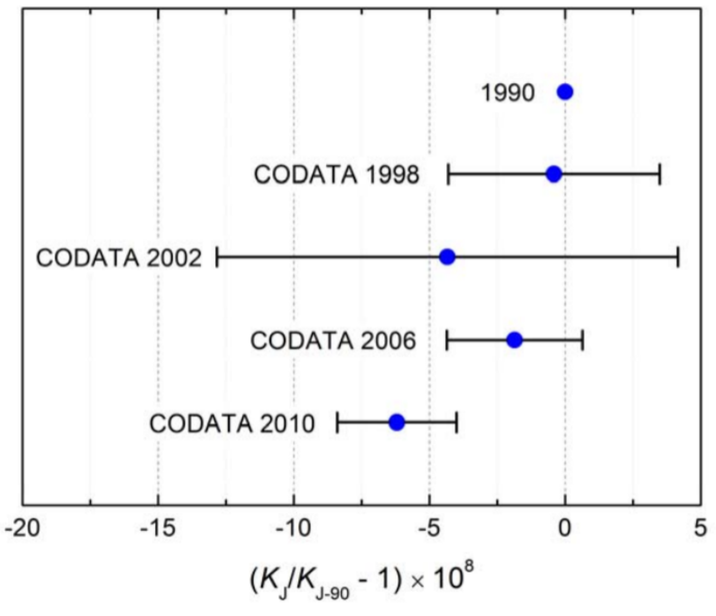
Translation: The ampere will be defined in terms of how many elementary electrical charges pass per second instead of by an *imaginary and impossible experiment* involving the force between two infinite parallel, current-carrying wires.

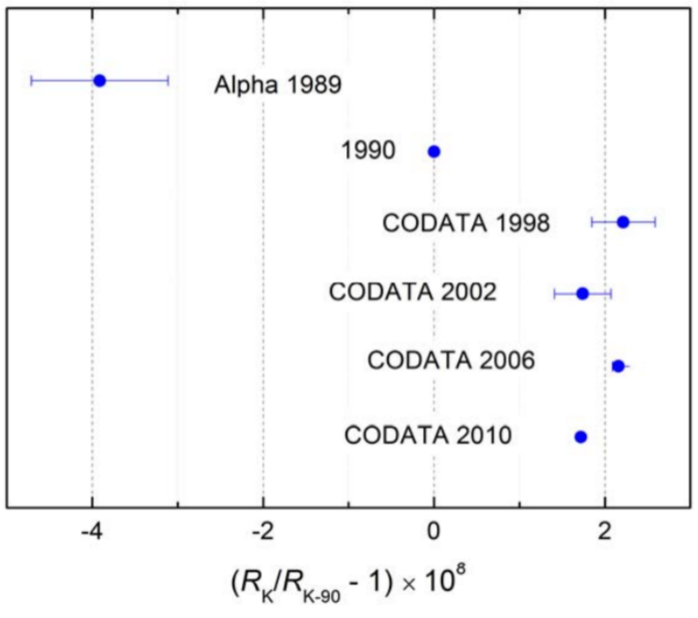
Effect of the new Ampere

- Excellent reproducibility has underpinned the worldwide uniformity of electrical units since 1990.

$$K_{J-90} = 2e/h \equiv 483\ 597.9\ \text{GHz/V}$$

$$R_{K-90} = h/e^2 \equiv 25\ 812.807\ \Omega$$





Effect of the new Ampere

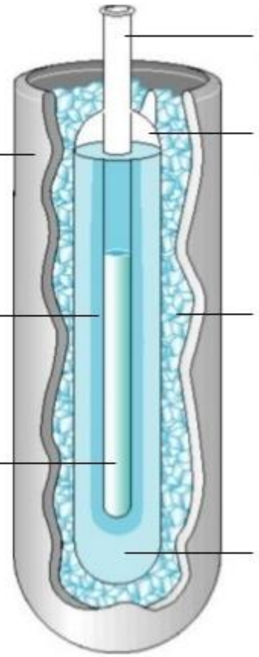
- When the 1990 values are replaced, small step changes are inevitable

The relative change from $K_{J\text{-}90}$ to K_J will be of the order 1×10^{-7}

The relative change from $R_{K\text{-}90}$ to R_K will be of the order 2×10^{-8}

- The changes should only be visible to labs operating primary quantum standards; calibrations of even the most stable standard resistors and Zener references should be largely unaffected

Kelvin

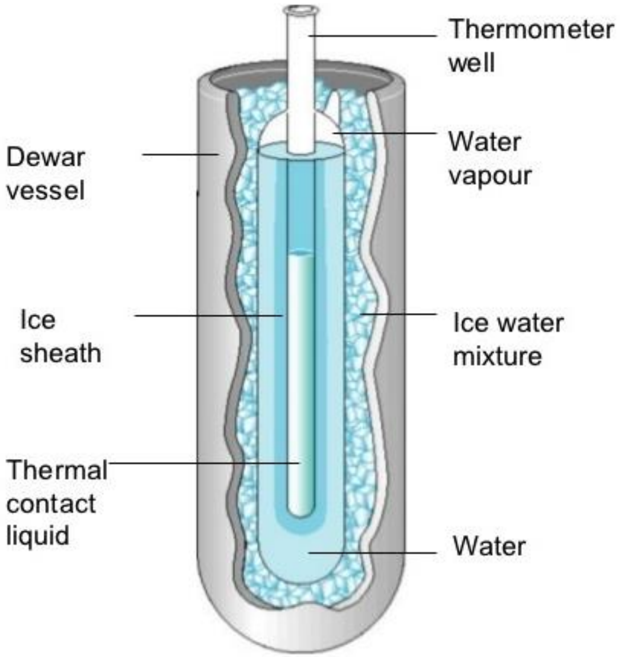


Old:

The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

- Not any water, but only Vienna Standard Mean Ocean Water.
- Not everbody and everywater can produce the same Kelvin
- The name does not refer to seawater, but to the chemical composition of distilled ocean water. “Vienna” is part of the name because the International Atomic Energy Agency, based in Vienna, promulgated the standard formulation. It is “defined exactly by the following amount of substance ratios: 0.00015576 mole of 2H per mole of 1H; 0.0003799 mole of 17O per mole of 16O, and 0.0020052 mole of 18O per mole of 16O.”

Defined only one point

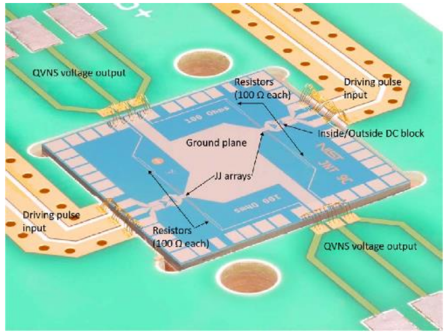


From Zigya

From NPL

Acoustic Gas Thermometry

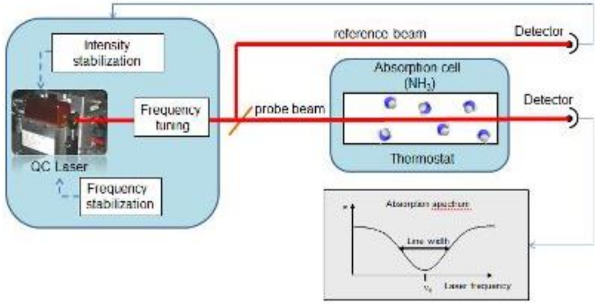
$$\mu_0{}^2 = \gamma \frac{k_B T}{m}$$



From NIST

Johnson Noise Thermometry

$$\bar{V}^2 = 4k_B T R \Delta f$$

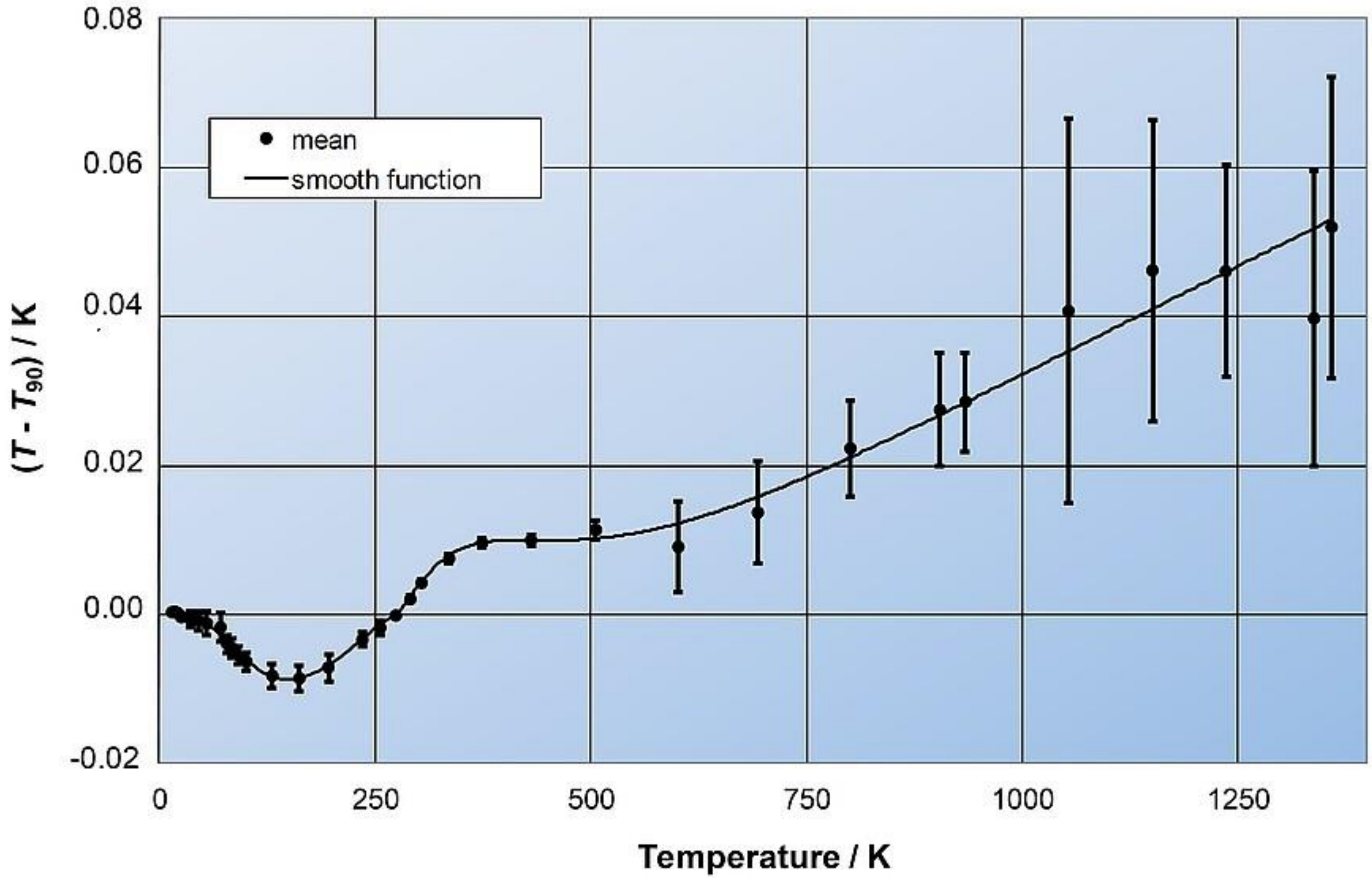


From LPL

Doppler Broadening Thermometry

$$\Delta v_D = \frac{v_0}{c}\sqrt{2ln2\frac{k_B T}{M}}$$

Temperature problem



New definition : Boltzmann constant

The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant *k* to be $1.380\ 649 \times 10^{-23}$ when expressed in the unit $J \cdot K^{-1}$, which is equal to $kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}$, where the kilogram, metre and second are defined in terms of *h*, *c* and $\Delta\nu_{Cs}$.

Translation: The kelvin will be defined through the constant relating thermodynamic temperature to energy (Boltzmann's constant), instead of by the point at which water coexists as a liquid, gas and solid.

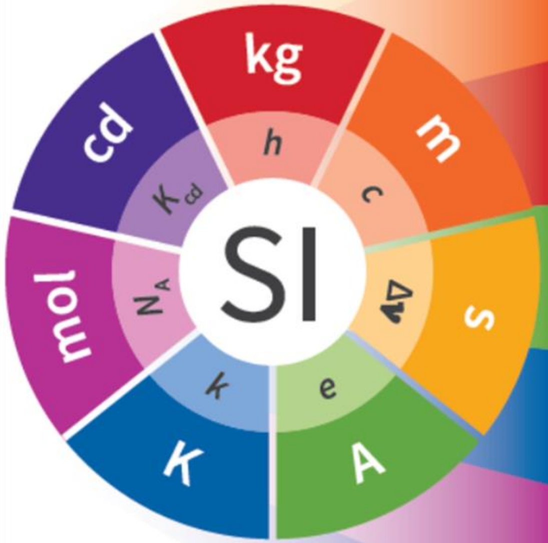
Acceptance : For the redefinition of the kelvin, the relative uncertainty of Boltzmann constant derived from two fundamentally different methods such as acoustic gas thermometry and dielectric constant gas thermometry be better than 10^{-6} and that these values be corroborated by other measurements.

Define the SI

- Fundamental Constant
  - Length: c
- Conventions
  - Time: $\nu(^{133}Cs)$
  - Temperature: T_{TPW}
  - Mass: M_{IPK}
- Conversion Factors
  - Electric Current: μ_0
  - Amount of Substance: $M(^{12}C)$
  - Luminous Intensity: K_{cd}

- Fundamental Constant
  - c: Length
  - h: Mass
  - e: Electric Current
  - k_B: Temperature
  - N_A: Amount of Substance
  - K_{cd}: Luminous Intensity
- Material Property
  - $\nu(\text{hfs } ^{133}Cs)$: Time

From Martin Milton's slide



Second (s)	$\Delta\nu_{Cs}$	**9 192 631 770**	**Hz**
Meter (m)	c	**299 792 458**	**m/s**
Kilogram (kg)	h	**6.626 070 15 x10^{-34}**	**J s**
Ampere (A)	e	**1.602 176 634 x10^{-19}**	**C**
Kelvin (K)	k	**1.380 649 x10^{-23}**	**J/K**
Mole (mol)	N_A	**6.022 140 76 x10^{23}**	**mol^{-1}**
Candela (cd)	K_{cd}	**683**	**lm/W**

The rules of nature to create the rules of measurement