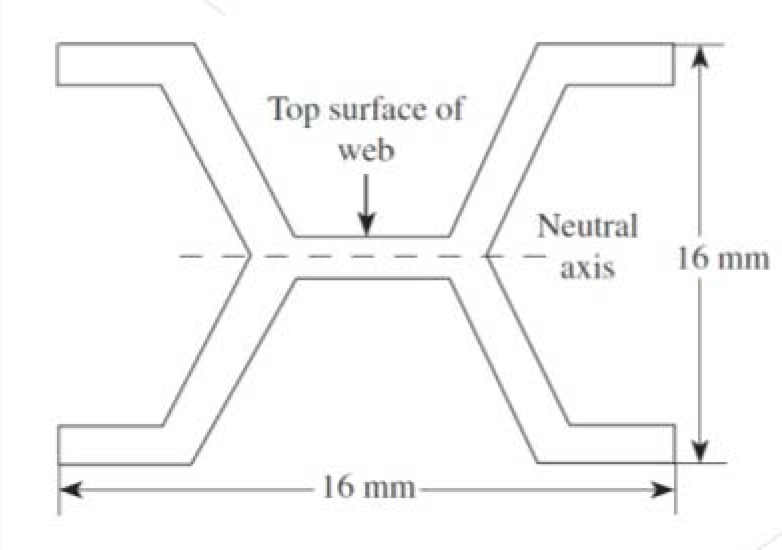
เมตร - METRE

ดร.จริยา บัวเจริญ

ฝ่ายมาตรวิทยามิติ

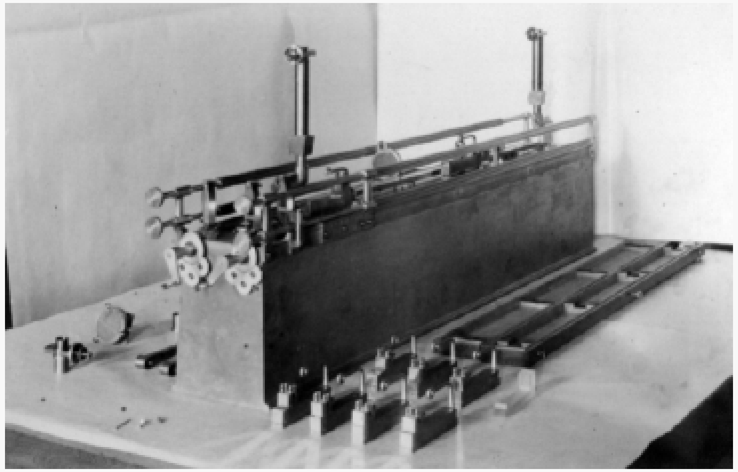
Original Metre

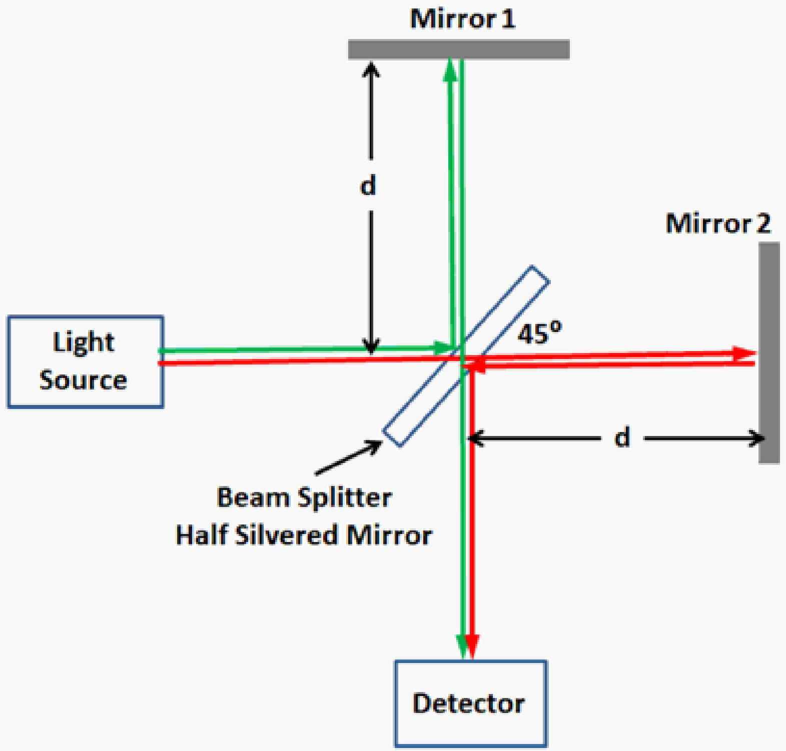
- The 1889 definition of the metre was based on the international prototype of platinum-iridium.



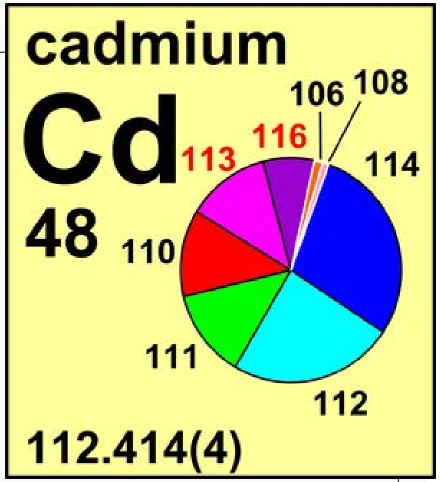
Invention of Interferometer

- The first interferometric measurements carried out using the international prototype metre were those of Albert A. Michelson and Jean-René Benoît (1892–1893) and of Benoît, Fabry and Perot (1906), both using the red line of cadmium.



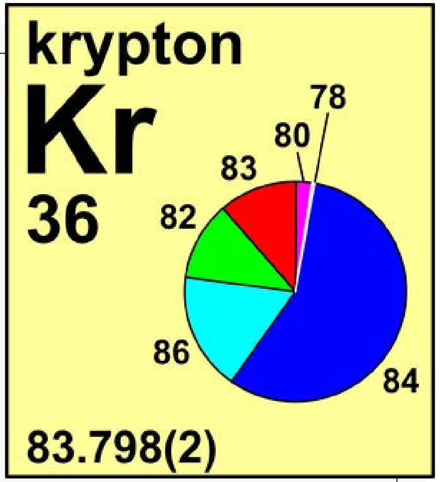


ångström



- These results, which gave the wavelength of the cadmium line (**λ ≈ 6438.47 angstroms**), led to the definition of the ångström as a secondary unit of length for spectroscopic measurements, first by the International Union for Cooperation in Solar Research (1907) and later by the CIPM (1927).
- The solution was **to define the metre in the same manner as the ångström had been defined in 1907**, that is in terms of the best interferometric wavelength available. Advances in both experimental technique and theory showed that the cadmium line was actually a cluster of closely separated lines, and that this was due to the presence of different isotopes in natural cadmium (eight in total). To get the most precisely defined line, it was necessary **to use a monoisotopic source** and this source should contain an isotope with even numbers of protons and neutrons (so as to have zero nuclear spin).

1^{st} revision of Metre

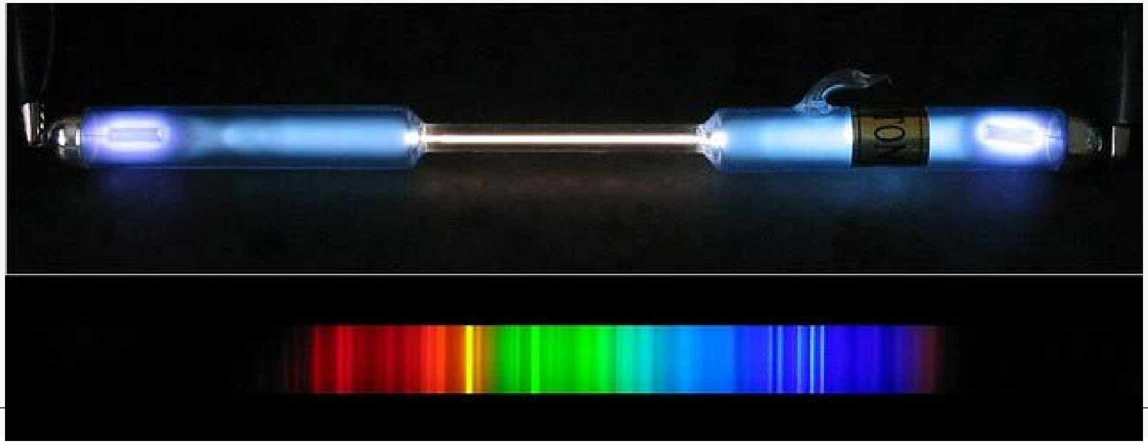


- Krypton is a gas at room temperature, allowing for easier isotopic enrichment and lower operating temperatures for the lamp (which reduces broadening of the line due to the Doppler effect), and so it was decided to select the **orange line of krypton-86 (λ = 605.780 210 3 nm)** as the new wavelength standard.
- **Accordingly, the 11th CGPM in 1960 agreed a new definition of the metre:**
- **The metre is the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton 86 atom.**
- The measurement of the wavelength of the krypton line was not made directly against the international prototype metre; instead, the ratio of the wavelength of the krypton line to that of the cadmium line was determined in vacuum. This was then compared to the 1906 Fabry–Perot determination of the wavelength of the cadmium line in air (with a correction for the refractive index of air. In this way, the new definition of the metre was **traceable to both the old prototype metre and the old definition of the ångström.**
- This also obsoleted the 1927 definition of the ångström based on the red cadmium spectral line, replacing it with **1 Å = 10−10 m.**

https://www.bipm.org/utils/common/pdf/mep/M-e-P_Kr_606.pdf

Invention of Laser

- The krypton-86 discharge lamp was superseded by a new invention: the laser, of which the first working version was constructed in the same year as the redefinition of the metre. Laser light is usually **highly monochromatic**, and is also **coherent** (all the light has the same phase, unlike the light from a discharge lamp), both of which are advantageous for interferometry.
- The shortcomings of the krypton standard were demonstrated by the measurement of the wavelength of the light from a methane-stabilized helium–neon laser ($\lambda \approx 3.39$ μm). **The krypton line was found to be asymmetrical**, so different wavelengths could be found for the laser light depending on which point on the krypton line was taken for reference. The asymmetry also affected the precision to which the wavelengths could be measured.



From prototype to physical constant

- Developments in electronics also made it possible for the first time to measure the frequency of light in or near the visible region of the spectrum, instead of inferring the frequency from the wavelength and the speed of light. Although visible and infrared frequencies were still too high to be directly measured, it was possible to construct a "chain" of laser frequencies that, by suitable multiplication, differ from each other by only **a directly measurable frequency in the microwave region**. The frequency of the light from the methane-stabilized laser was found to be 88.376 181 627(50) THz.

$$c = f \cdot \lambda$$

- Independent measurements of frequency and wavelength are a measurement of the speed of light, and the results from the methane-stabilized laser gave the value for the speed of light with an uncertainty almost 100 times lower than previous measurements in the microwave region. Or, somewhat inconveniently, the results gave two values for the speed of light, depending on which point on the krypton line was chosen to define the metre. This ambiguity was resolved in 1975, when **the 15th CGPM approved a conventional value of the speed of light as exactly 299 792 458 m s^{-1}**.

Metre as today

- This change was adopted in order to improve the accuracy with which the definition of the metre could be realized, the realization being achieved using an interferometer with a travelling microscope to measure the optical path difference as the fringes were counted. In turn, this was replaced in 1983 by the 17th CGPM (1983, Resolution 1) that specified the current definition, as follows:
- **The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.**
- It follows that the speed of light in vacuum is exactly 299 792 458 metres per second, c_0 = 299 792 458 m/s.

Definition of the Metre since 1798

Basis of definition	**Date**	**Relative uncertainty**
1/10,000,000 part of one half of a meridian, measurement by Delambre and Mechain	1798	10^{-4}
1st prototype Metre des Achchives platinum bar standard	1799	10^{-5}
Platinum-iridium bar at melting point of ice (1st CGPM)	1889	10^{-7}
Platinum-iridium bar at melting point of ice, atmospheric pressure, supported by 2 rollers (7th CGPM)	1927	10^{-7}
1,650,763.73 wavelengths of light from a specified transition in krypton-86 (11th CGPM)	1960	10^{-8}
Length of the part travelled by light in a vacuum in 1/299,792,458 of a second (17th CGPM)	1983	10^{-10}

MeP of metre

- Primary methods for the practical realization of the definition of the metre.
- The fundamental equation underlying the definition of the metre is a direct relationship between a length, a time interval and the speed of light:

$$l = c \cdot \Delta t$$

- in which c is the fixed value for the speed of light in vacuum, c = 299 792 458 m s^{-1} , and Δt is the travelling time of the light along a geometrical path, of length l.

- Accordingly, the definition of the metre can be realized in practice by one of the following primary methods:
  a) by **direct measurement** of light travelling time,
  b) by **indirect measurement** of light travelling time.

Direct measurement (time of flight measurement, tof)

- The two pulses must be clearly separated in time.

$$l = \Delta z = \frac{1}{2} \cdot c_g \cdot \Delta t$$

- in which c_g is the group velocity of the wave packet.
- **Suitable for long range.**

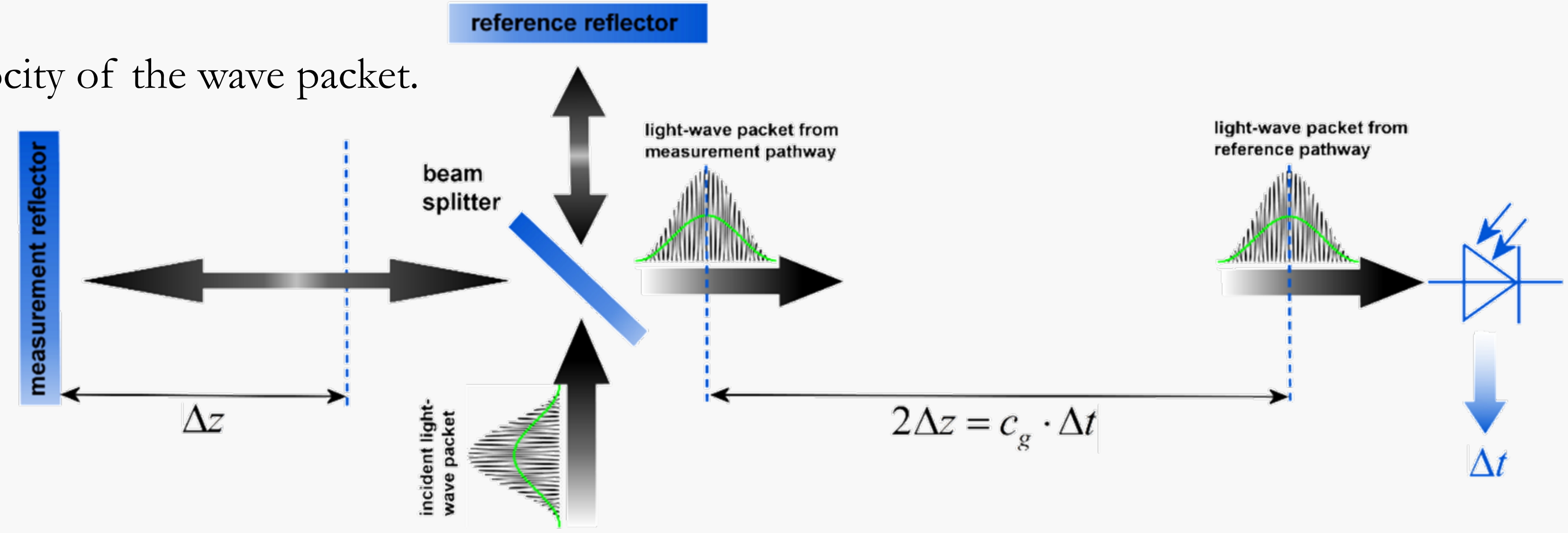
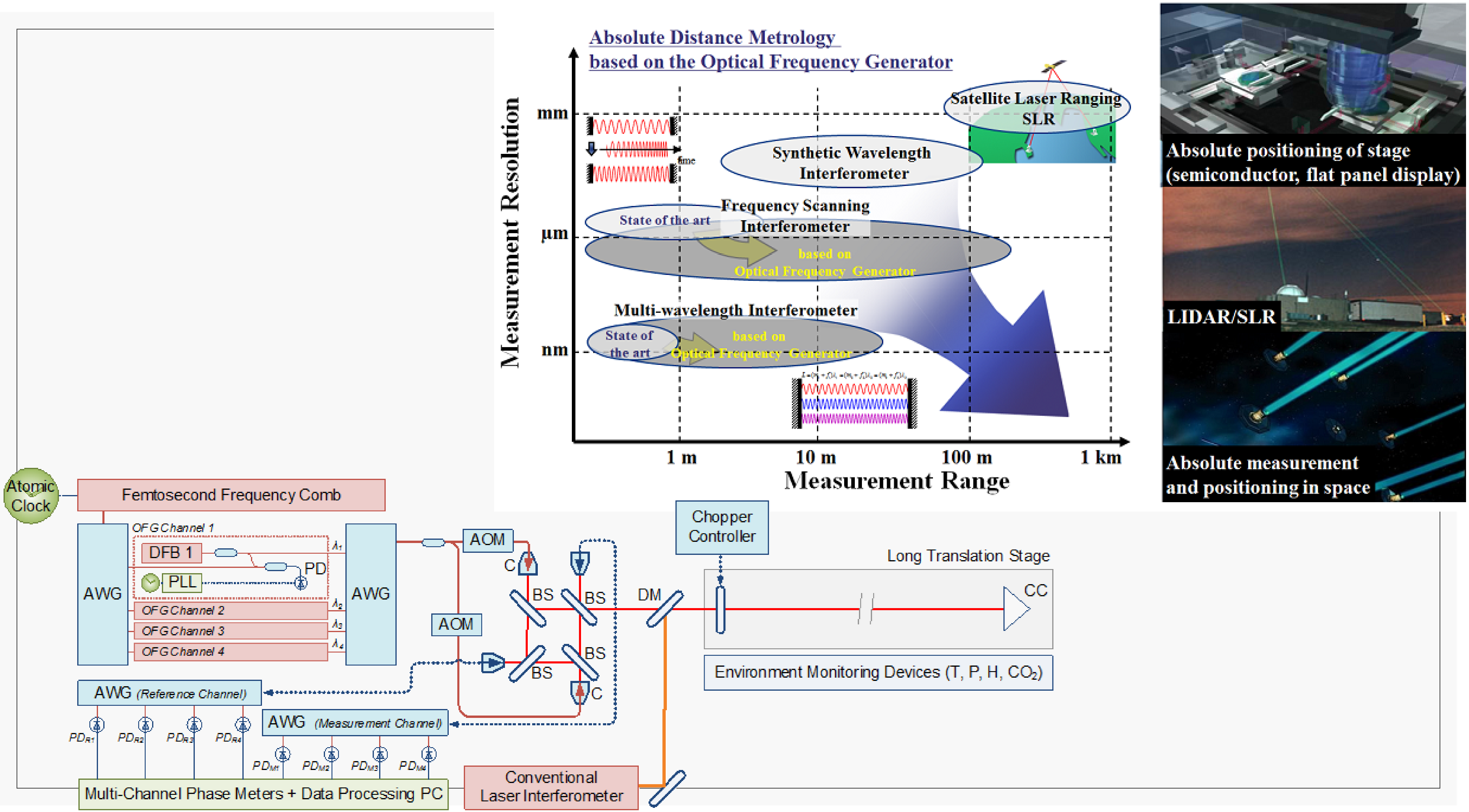


Figure 1. Primary realization of the length by direct measurement of time delay between light-wave packets travelling pathways of different lengths before reaching a detector. The green curves inside the wave packets indicate the average light intensity that is sensed by the detector.


Absolute Distance Metrology
based on the Optical Frequency Generator
Measurement Resolution
mm
μm
nm
Satellite Laser Ranging
SLR
Synthetic Wavelength
Interferometer
Frequency Scanning
Interferometer
State of the art
based on
Optical Frequency Generator
Multi-wavelength Interferometer
State of
the art
based on
Optical Frequency Generator
1 m
10 m
100 m
1 km
Measurement Range
Absolute positioning of stage
(semiconductor, flat panel display)
LIDAR/SLR
Absolute measurement
and positioning in space
Atomic
Clock
Femtosecond Frequency Comb
AWG
OFG Channel 1
DFB 1
PD
PLL
OFG Channel 2
OFG Channel 3
OFG Channel 4
AWG
AOM
AOM
C
BS
BS
BS
BS
C
DM
Chopper
Controller
Long Translation Stage
CC
Environment Monitoring Devices (T, P, H, CO_2)
AWG (Reference Channel)
AWG (Measurement Channel)
Multi-Channel Phase Meters + Data Processing PC
Conventional
Laser Interferometer

Indirect measurement (optical interferometry)

- **Suitable for short range (a few metres)**
- While the average intensity of a single monochromatic light wave is just related to the square of its amplitude, interference of two light waves of the same frequency results in a detectable intensity:

$$I = I_0(1 + \gamma \cdot cos[\varphi_1 - \varphi_2])$$

$$l = \Delta z = \frac{\lambda}{2} \cdot \frac{\Delta\varphi}{2\pi}$$

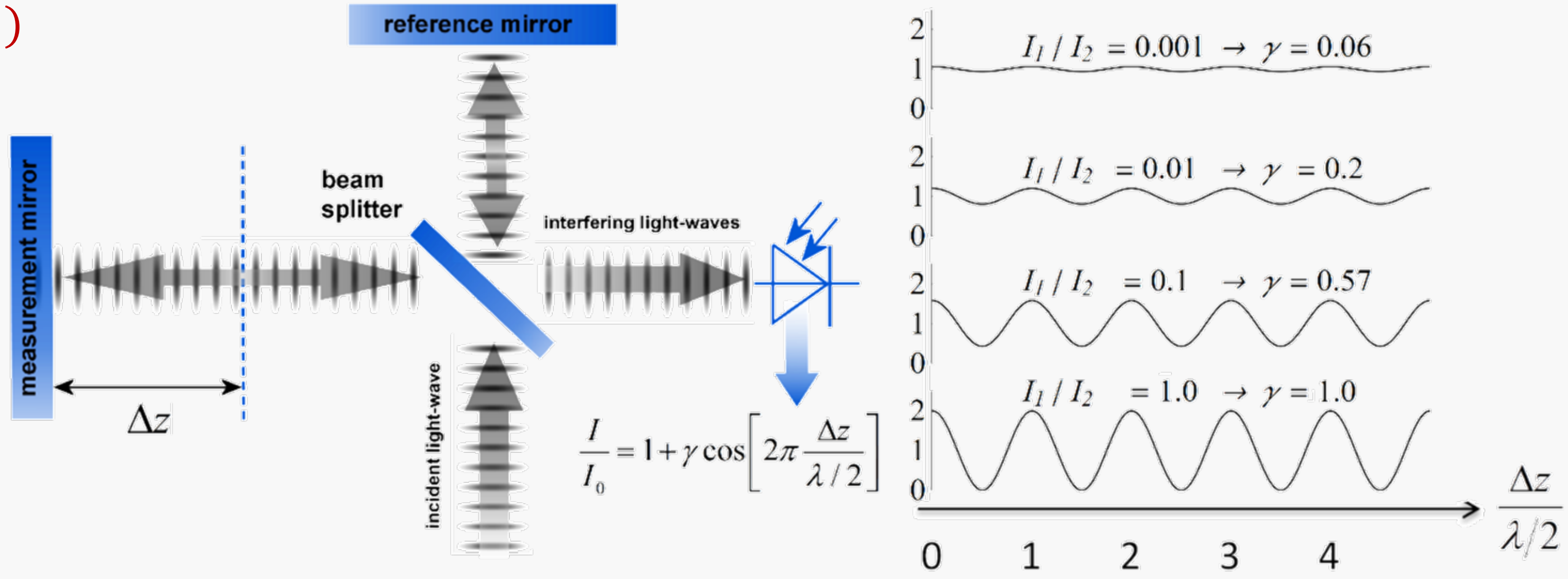
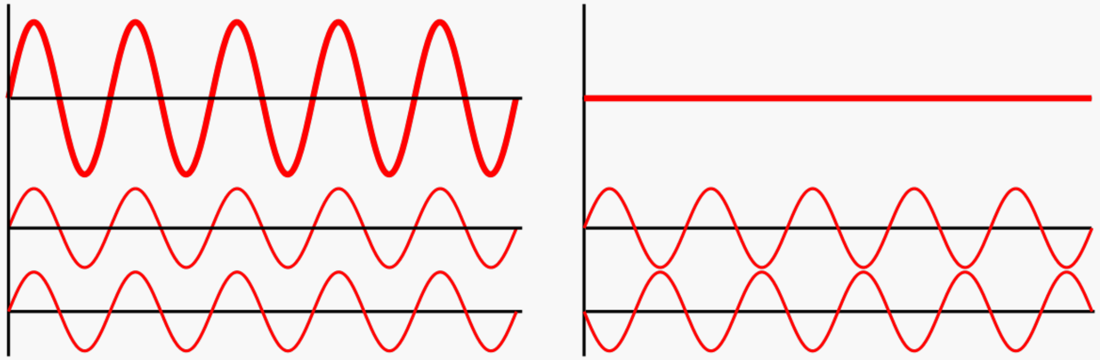
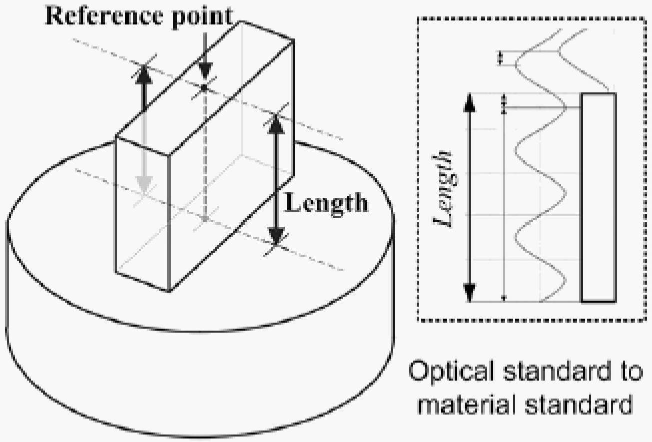


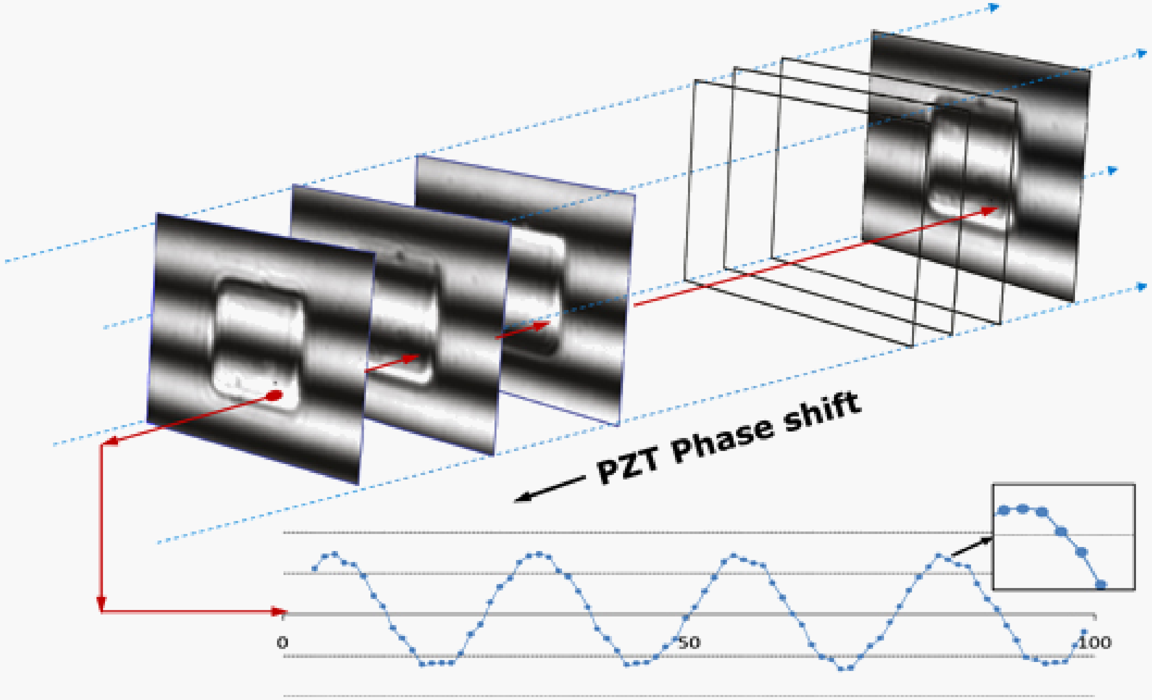
Figure 2. Primary realization of the length unit by interferometry, i.e. by indirect measurement of the time delay between monochromatic light-waves travelling pathways of different lengths before reaching a detector.

Interferometry



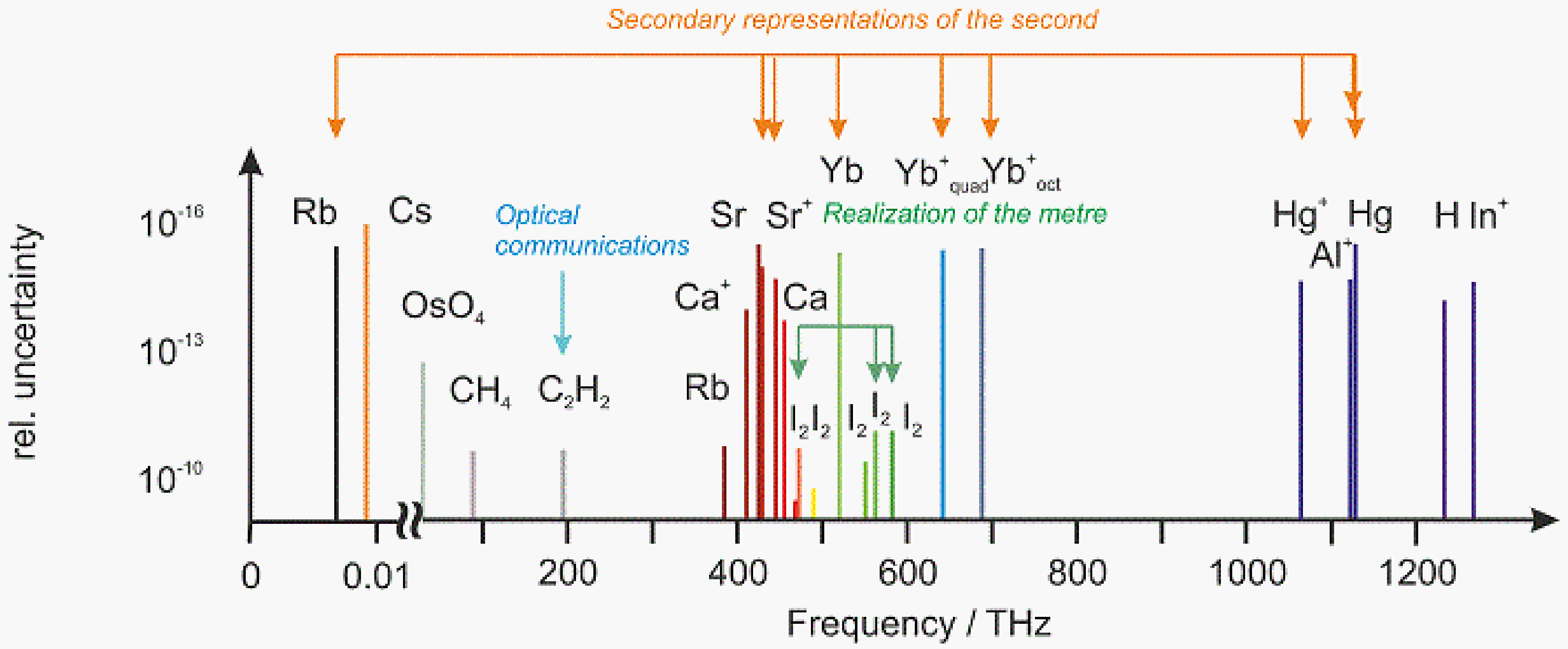


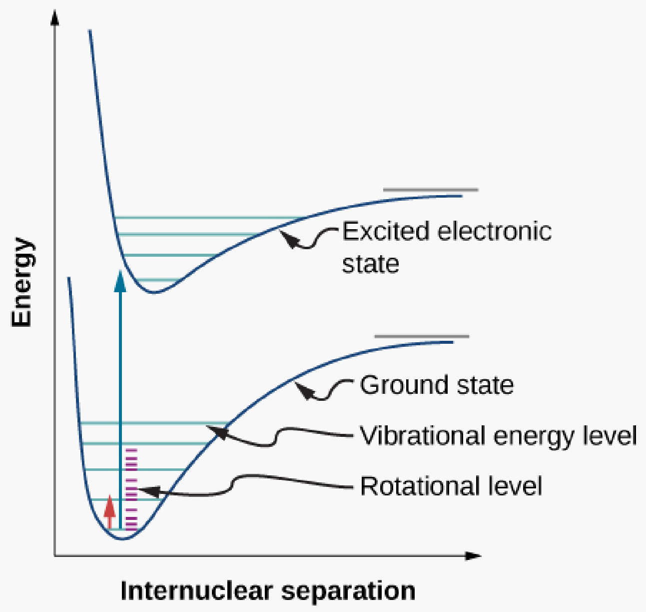
Optical standard to material standard

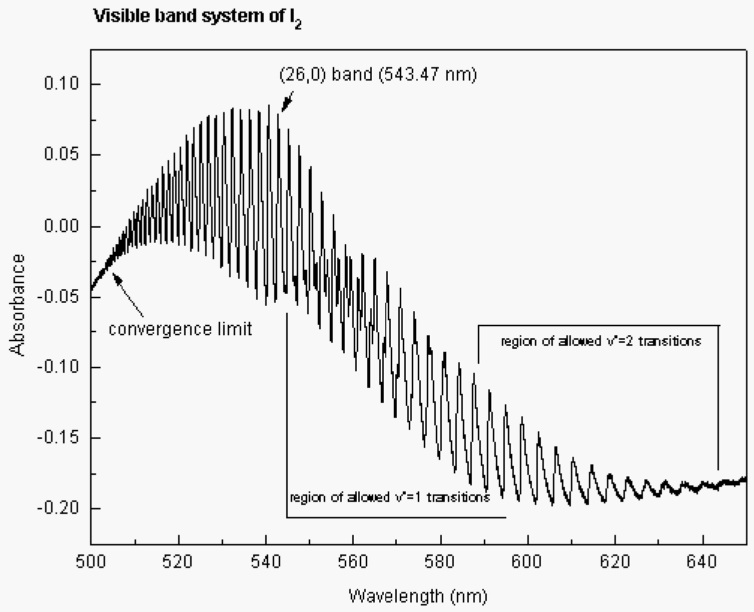


Recommended values of standard frequencies

- The infrared light from a methane-stabilized laser was inconvenient for use in practical interferometry. It was not until 1983 that the chain of frequency measurements reached the 633 nm line of the He-Ne laser, stabilized using molecular iodine.







https://www.bipm.org/en/publications/mises-en-pratique/standard-frequencies.html

Secondary methods for Dimensional Nanometrology

- At scales relevant to current dimensional nanometrology, the primary method is **limited by fringe sub-division and periodic non-linearities in visible-wavelength interferometry**.
- Nano-scale manufacturing requires the better accuracy levels, at the nanometre or **sub-nanometre scale** for which the traceability infrastructure is not fully available. Thus, an alternative route to traceability at the nanometre and sub- nanometre level is necessary.
- The success of the semiconductor industry and prevalence of silicon-based technology has led to silicon being one of the most thoroughly studied materials in nature and the availability of very high purity crystalline silicon. Work in preparation for the 2018 revision of the SI, has resulted in an agreed CODATA value **for the Si {220} lattice spacing d_{220} = 192.015 571 4 × 10^{-12} m, with a standard uncertainty of 0.000 003 2 × 10^{-12} m**, (i.e. $\Delta d/d$ = 1.67 × 10-8) at a temperature of 22.5 °C in vacuum. This is the lattice spacing of an ideal single crystal of natural-isotopical undoped silicon that is free of impurities and imperfections.

Impurities vs uncertainty

- Impurities and vacancies affect the lattice parameter. To achieve values of lattice spacing uncertainty approaching the CODATA value, the concentration of the impurities in a silicon crystal must be determined either from the manufacturer or by using a suitable technique such as, X-ray fluorescence, neutron activation, infra-red or mass spectroscopy.

Atom	Theoretical Radius (nm)	Experimental radius (nm)	Theoretical strain parameter β (10^{-24} cm^{-3})	Experimental strain parameter β (10^{-24} cm^{-3})
C	0.077	0.077	-6.9	-6.9 ± 0.2
O	interstitial	0.142		+4.4 ± 0.5
N	interstitial	0.150		+5.7 ± 0.1
B	0.088	0.084	-5.1	-5.6 ± 0.2
P	0.110	0.109	-1.4	-1.3 ± 0.2
As	0.118	0.117	±0	-0.007 ± 0.5
Sb	0.136	0.133	+3	+2.8 ± 0.2
Vacancies	0.129	0.1274	+2	+1.7 ± 0.5
Si	0.117	0.1176		

Table 4.1 Reproduced from [4.1] showing theoretical and experimental values for the effective radius and lattice strain parameters (β) of impurity atoms in a silicon lattice.

Traceability for Dimensional Nanometrology

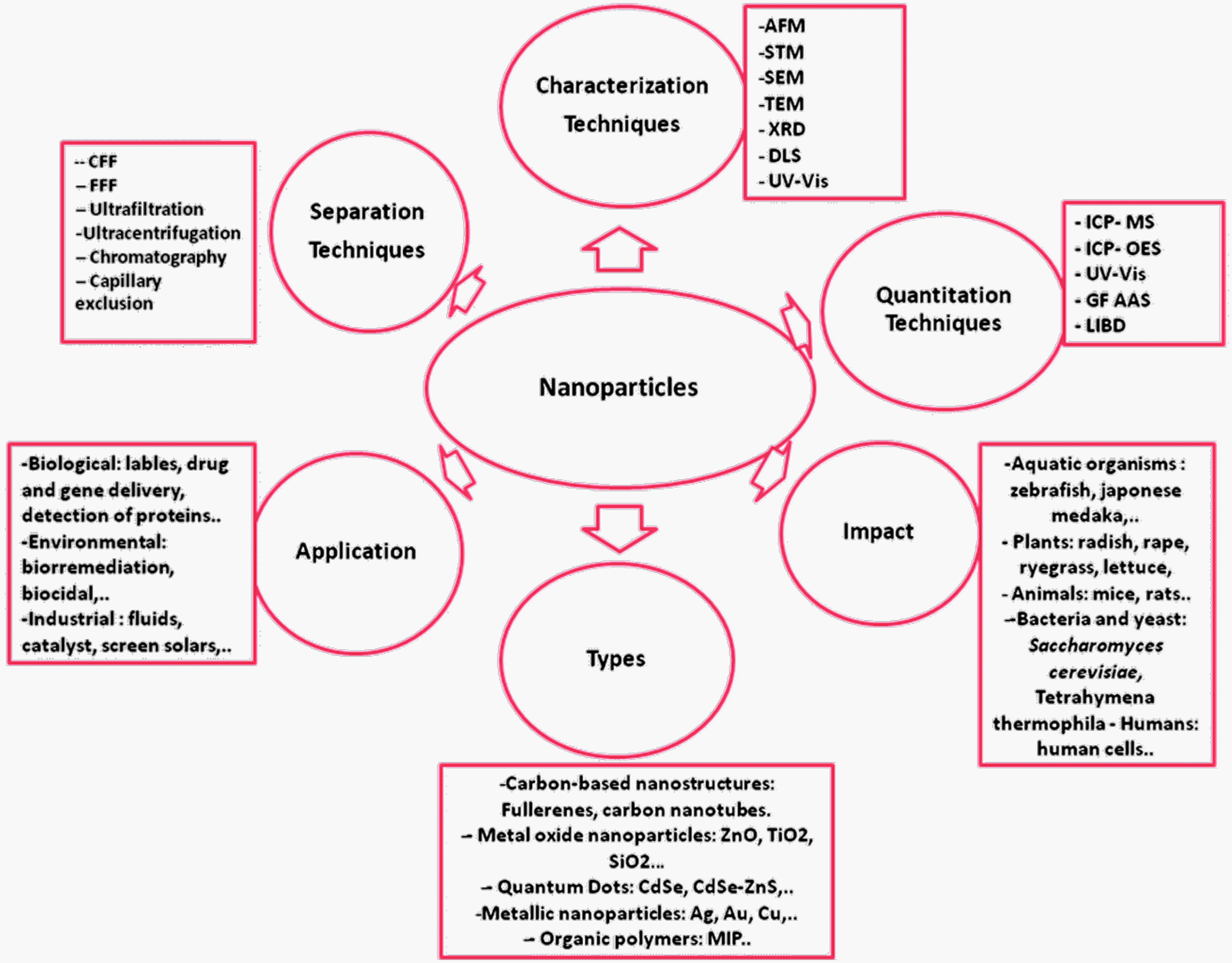
- There is a variety of examples of how a traceability pathway through the silicon lattice spacing is relevant for dimensional nanometrology. Three of these are particularly noteworthy:

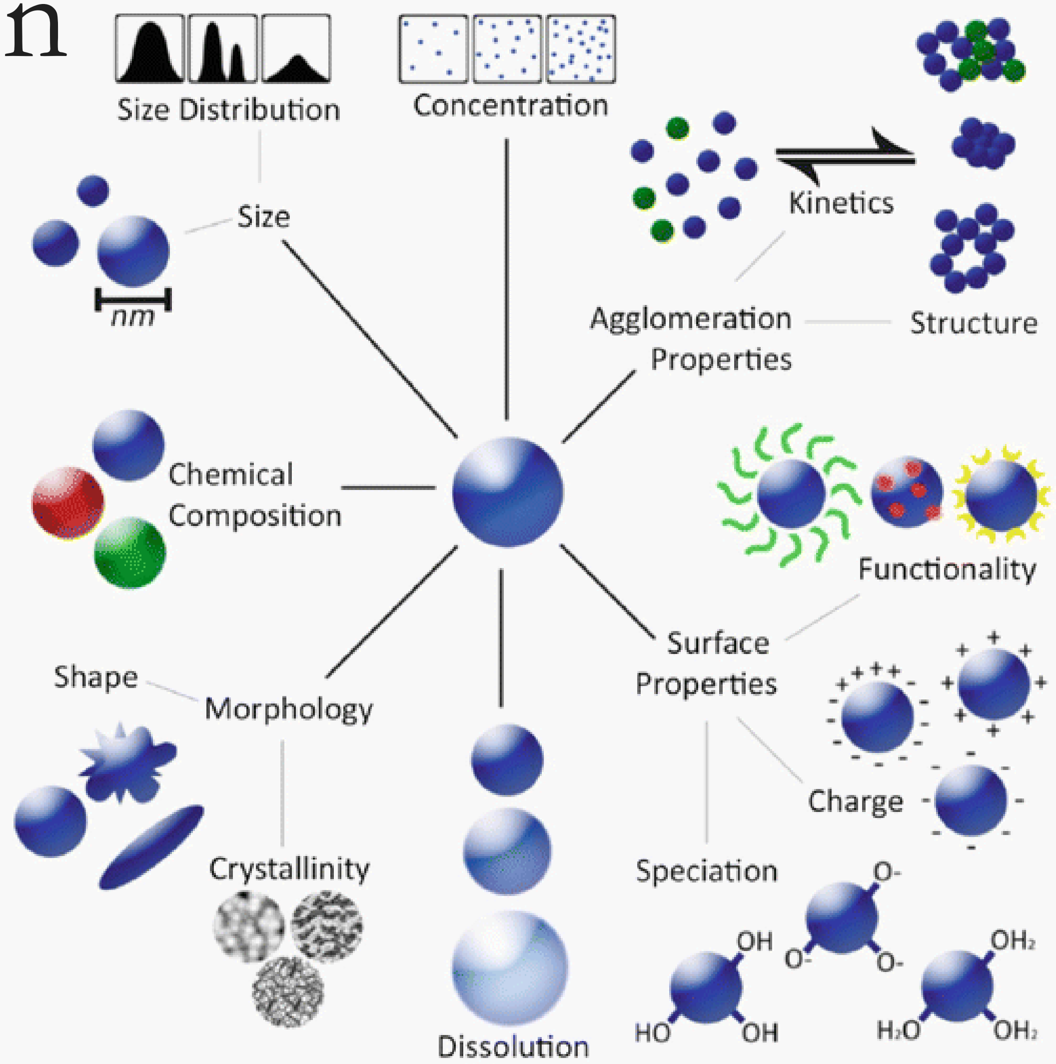
(1) Measurement of a displacement by reference to the **d_{220} lattice plane**, using an **X-ray interferometry**. An uncertainty of 10 pm is realistic with a 10 μm displacement from a monolithic interferometer.

(2) Calibration of **TEM** magnification by reference to a single crystal silicon artefact. By this method expanded uncertainties below 1 nm for the widths of line structures smaller than 200 nm could be achieved.

(3) Measurement of **step height standard artefacts** manufactured from single crystal silicon, where the height range of multiple monoatomic steps currently is limited up to 10 nm and the uncertainties of the monoatomic step heights are 5 pm under UHV conditions and 15 pm under ambient conditions.

Material characterization





ขอบคุณค่ะ